مجلة روافـــد المعرفـــــة

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ترسل البحوث وجميع المراسلات المتعلقة بالمجلة إلى العنوان التالي:

كلية العلـــوم – جامعـة الزيتونــة – ترهـــونة

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شروط وتعليمات النشر

1- أن يكون البحث أصيلاً ومبتكراً ولم يسبق نشره في أي جهة أخرى، وتتوفر فيه شروط البحث العلمي المعتمدة على الأصول العلمية والمنهجية المتعارف عليها في كتابة البحوث الاكاديمية.
2- أن يكون البحث مكتوباً بلغة سليمة، ومراعياً لقواعد الضبط ودقة الرسوم والاشكال – إن وجدت – ومطبوعاً ببنط (14) وبخط (Simplified Arabic)، وألا تزيد صفحات البحث عن (35) صفحة متضمنة الهوامش والمراجع.
3- يجب أن يشتمل البحث على العناصر التالية: - عنوان البحث باللغتين العربية والإنجليزية؛ - ملخص تنفيذي باللغتين العربية والإنجليزية في نحو 100-125 كلمة، والكلمات المفتاحية (keywords) بعد الملخص.
4- يتم توثيق الهوامش وفق طريقة **APA** (طريقة الجمعية الأمريكية السيكولوجية**)** بإصدارتها المختلفة.
5- يُفضل أن تكون الجداول والاشكال مدرجة في أماكنها الصحيحة، وأن تشمل العناوين والبيانات الإيضاحية الضرورية، ويراعى ألا تتجاوز أبعاد الاشكال والجداول حجم حيز الكتابة في صفحة Microsoft Word.
6- أن يكون البحث ملتزماً بدقة التوثيق، وحسن استخدام المصادر والمراجع، وأن تثبت مصادر ومراجع البحث في نهاية البحث.
7- تحتفظ المجلة بحقها في اخراج البحث وإبراز عناوينه بما يتناسب واسلوبها في النشر.
8- ترحب المجلة بنشر البحوث المكتوبة باللغة الأجنبية ويفضل أن يرفق البحث بملخص باللغة العربية (لا يتجاوز 200 كلمة).
9- ترحب المجلة بنشر ما يصلها من ملخصات الرسائل الجامعية التي تمت مناقشتها وإجازتها، على أن يكون الملخص من إعداد صاحب الرسالة نفسه.
10- تُرسل نسخة من البحث مطبوعة على ورق بحجم (A4) إلى مقر المجلة، ونسخة إلكترونية إلى إيميل المجلة: rwafedalmarefa@gmail.com، على أن يدون على صفحة الغلاف: اسم الباحث، لقبه العلمي، مكان عمله، تخصصه، رقم هاتفه وبريده الإلكتروني.
11- يخطر الباحث بقرار صلاحية بحثه للنشر من عدمها خلال مدة ثلاثة أشهر من تاريخ استلام البحث.
12- في حالة ورود ملاحظات وتعديلات على البحث من المحكم، ترسل تلك الملاحظات إلى الباحث لإجراء التعديلات اللازمة بموجبها، على أن تعاد للمجلة خلال مدة أقصاها شهر واحد.
13- الأبحاث التي لم تتم الموافقة على نشرها لا تعاد إلى الباحثين.
14- تؤول جميع حقوق النشر للمجلة.
15- دفع رسوم التحكيم العلمي والمراجعة اللغوية والنشر، إن وجدت.

البحوث المنشورة في هذه المجلة تعبر عن رأي أصحابها ولا تعبر بالضرورة عن رأي المجلة أو الجامعة.

الكلمة الافتتاحية

بسم الله الرحمن الرحيم، عليه نتوكل وبه نستعين، نحمده سبحانه كما ينبغي أن يُحمد، ونصلي ونسلم على رسوله محمد وعلى آله وصحبه والتابعين.

وبعـــد،،،

إن سبيل نهضة الأمم إنما يكون بالبحث العلمي في شتى المجالات، فدوره مهم لمواكبة التقدم والرقي بالمجتمع فبالبحث العلمي ينمى القدرات البشرية وهو الأساس في الابتكاروالإبداع. بعون من الله وتوفيقه، وبعد الجهد الكبير الذي بذلته هيئة التّحرير تكاملت الاستعدادات لإصدار العدد التاسع من مجلة رو افد المعرفة، والذي نأمل أن يلبي طموحات المهتمين والباحثين. ومن هنا ندعو كل الباحثين والكتاب الإسهام في استمرار المجلة بتقديم نتاجهم العلمي للنشر، ونرحب بآراء القراء والباحثين ونقدهم البناء حتى تخرج المجلة في صورتها المثلى وليكون العدد التالي أفضل من سابقه. وختاما يجدر بنا مع إصدار هذا العدد والذي يحتوي على عدد أربعة عشر بحثاً أصيلاً مختلفاً، أن نتقدم بجزيل الشكر والتقدير للمحكمين والمؤلفين وكل من أسهم في إخراجه وتصميمه، آملين أن تكون محتوياته نافعة للجميع.

والحمد لله في بدءٍ ومُخْتَتَمِ.

هيئة التحرير

.

المحتويات

Spectra of the Upper Triangular Double-Band Matrix $\triangle^{ab}$ on the Sequence Space bs

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Abstract:

Many authors have investigated the fine spectrum of the generalized difference operator on different sequence spaces. Recently, spectra of the operator $\triangle_{ab}$ on the sequence space bv_0 has been determined. Our plan in this work is to get new results associated with the spectra of the operator $\triangle^{ab}$ on the sequence space bs by using the results in[4]. Moreover, the purpose of this work is to study a wider class of operators on specific space which has not been covered before in the literature.

Keywords: Spectrum, Infinite matrices, Sequence spaces.

الملخص

العديد من الباحثين قد استنبطوا الطيف والطيف الدقيق لمؤثر الفرق المعمم على فراغات متتابعات مختلفة. حديثاً الطيف و الطيف الدقيق لمؤثر الفرق المعمم $\triangle_{ab}$ على فراغ المتتابعات bv_0 ، تم تحديده (انظر المرجع [4]). مخططنا في هذا العمل إيجاد نتائج جديدة مرتبطة بالطيف والطيف الدقيق لمؤثر الفرق المعمم $\triangle^{ab}$ على فراغ المتتابعات bs باستخدام النتائج الموجودة بالمرجع[4]. بالإضافة لذلك الهدف من هذا العمل تقديم دراسة مختلفة للطيف لمؤثر الفرق العلوي المعمم على فراغ معين التي وجدنا أنها لم تُغطى سابقاً، حيث أن هذا الفراغ المهم لم تدرس عليه مسألة تحديد الطيف والطيف الدقيق للمؤثر العلوي.

الكلمات المفتاحية: الطيف، المصفوفات اللانهائية البعد، فراغات المتتابعات.

1. Review of the literature and well-known results

The fine structure of the spectrum of the upper triangular double-band matrices have been studied in some special cases. For example; in 2010, the fine spectrum of upper triangular double-band matrix $U(r, s)$ as operator on the sequence spaces c_0 and c was studied by Karakaya and Altun [8]. Recently, El-Shabrawy and Abu-Janah[3] determined Spectra of the operator $\triangle^{ab}$ on the sequence space ℓ_p, where $(1 \le p < \infty)$. Also, see[2,5,6,7].

Now, we introduce some definitions and notations.

The set of all complex sequences is denoted by w, $\mathbb{C}$ denotes the complex field and $\mathbb{N}$ is the set of nonnegative integers.

We begin by giving the definitions of some sequence spaces, which are needed in this work.

Definition 1.1: The space bv_0 of all sequences of bounded variation is defined by

$$bv_0 = \{x = (x_k)_{k=0}^{\infty} \in w,\ x_k \in \mathbb{C}, \text{such that } \lim_{k\to\infty} x_k = 0,\ \sum_{k=0}^{\infty}|x_{k+1} - x_k| < \infty\}.$$

with the norm

$$\|x\|_{bv_0} = \sum_{k=0}^{\infty}|x_{k+1} - x_k|.$$

and the space bs of bounded series is defined by

$$bs = \{x = (x_k)_{k=0}^{\infty} \in w\colon\ x_k \in \mathbb{C},\ \sup_{n\in\mathbb{N}}|\sum_{k=0}^{n} x_k| < \infty\},$$

with norm

$$\|x\|_{bs} = \sup_{n\in\mathbb{N}}|\sum_{k=0}^{n} x_k|$$

bv_0 and bs are Banach spaces [10,15].

Definition 1.2: Let X be a complex Banach space, the continuous dual X^* of a sequence space X is defined as the set of all bounnded linear functionals on the space X.

It is well-known that $bv_0^* \cong bs$ [15].

Some basic concepts are required for our research and they will be given as follows.

Let X be a complex infinite dimensional Banach space and $B(X)$ be the set of all bounded linear operators on X into itself. If $\mathrm{T} \in \mathrm{B(X)}$, we use $\mathcal{R}(T)$ to denote the range of T. with T we associate the operator $T - \lambda I$.

The points λ in $\mathbb{C}$ are divided into two sets:

The resolvent set: $\rho(T,X) = \{\lambda \in \mathbb{C}\colon T - \lambda I \text{ is a bijection}\}$,

and $\mathbb{C}\backslash\rho(T,X)$,

The spectrum: $\sigma(T,X) = \{\lambda \in \mathbb{C}\colon T - \lambda I \text{ is not invertible}\}$,

$\sigma(T,X)$ can be analyzed into three disjoint sets as follows:

The point spectrum: $\sigma_p(T,X) = \{\lambda \in \mathbb{C}\colon T - \lambda I \text{ is not injective}\}, \lambda I \text{ is not injective}\}$

The continuous spectrum:

$$\sigma_c(T,X)=\{\lambda\in\mathbb{C}: T-\lambda I \text{ is injective and } \overline{\mathcal{R}(T-\lambda I)}=X, \text{but } \mathcal{R}(T-\lambda I)\neq X\},$$

The residual spectrum: $\sigma_r(T,X)=\{\lambda\in\mathbb{C}: T-\lambda I \text{ is injective, but } \overline{\mathcal{R}(T-\lambda I)}\neq X\}$.

$\sigma(T,X)=\sigma_p(T,X)\cup\sigma_c(T,X)\cup\sigma_p(T,X)$.

(see, Stone [11] or [9,10]).

A linear operator T with domain and range in a normed space X, is calssified I, II or III, according as its range, $\mathcal{R}(T)$. To understand that, see[12,13].

Three more subdivisions of the spectrum can be defined:

Definition 1.3: In what follows,

$\sigma_{\text{ap}}(T,X) = \{\lambda\epsilon\mathbb{C}: \text{there exists a Weyl sequence for } T-\lambda I\}$

is called the approximate point spectrum of T. (a sequence (x_k) in X is a *Weyl sequence* for T if $\|x_k\|=1$ and $\|Tx_k\|\to 0$, as $\kappa\to\infty$, $T\in B(X)$).

$\sigma_\delta(T,X)=\{\lambda\in\mathbb{C}: \mathcal{R}(\lambda I-T)\neq X\}$ is called defect spectrum of T.

And

$\sigma_{co}(T,X)=\{\lambda\in\mathbb{C}: \overline{\mathcal{R}(\lambda I-T)}\neq X\}$,

is often called compression spectrum.

$\sigma_{\text{ap}}(T,X)$ and $\sigma_\delta(T,X)$ are not necessarily disjoint. As well as $\sigma_{\text{ap}}(T,X)$ and $\sigma_{co}(T,X)$ are not necessarily disjoint.

Where

$$\sigma(T,X)=\sigma_{\text{ap}}(T,X)\cup\sigma_\delta(T,X),$$

$$\sigma(T,X)=\sigma_{\text{ap}}(T,X)\cup\sigma_{co}(T,X),$$

$$\sigma_{co}(T,X)\subseteq\sigma_\delta(T,X),$$

$$\sigma_p(T,X)\subseteq\sigma_{\text{ap}}(T,X),$$

$$\sigma(T,X)=\sigma_p(T,X)\cup\sigma_c(T,X)\cup\sigma_r(T,X)$$

we note that

$$\sigma_r(T,X)=\sigma_{co}(T,X)\setminus\sigma_p(T,X),$$

$$\sigma_r(T,X)=\sigma(T,X)\setminus[\sigma_p(T,X)\cup\sigma_{co}(T,X)],$$

$$\sigma_{\text{ap}}(T,X)=\sigma(T,X)\setminus III_1(T,X),$$

$$\sigma_\delta(T,X)=\sigma(T,X)\setminus I_3(T,X). \quad \textbf{(1.1)}$$

From Proposition in [1], we have

$$\sigma(T^*,X^*)=\sigma(T,X) \quad \textbf{(1.2)}$$

$$\sigma_p(T^*,X^*)=\sigma_{co}(T,X) \quad \textbf{(1.3)}$$

The following lemma will be used in the sequel.

Lemma 1.1. If T is a bounded linear operator on a Banach space X into itself, then

$$\sigma_r(T,X) = \sigma_p(T^*,X^*) \diagdown \sigma_p(T,X)$$

$\lambda \in \sigma_p(T^*,X^*) \diagdown \sigma_p(T,X)$), the operator $T-\lambda I$ is one to one and hence has an inverse. But $T^*-\lambda I$ is not one to one. Then, the range of the operator $T-\lambda I$ is not dense in X. So $\lambda \in \sigma_r(T,X)$.

2. Related work

The spectra of $\triangle_{ab}$ space bv_0 has been considered by El-Shabrawy and Abu-Janah in [4]. Here, we add some new results related with this work in [4].

To avoid repetition, we will state the following theorems which were proved in [4] without any details about the proofs.

The following theorem includes results about $\sigma(\triangle_{ab}, bv_0)$, $\sigma_p(\triangle_{ab}\, bv_0)$ and $\sigma_p({\triangle_{ab}}^*, {bv_0}^*)$.

Theorem 2.1.

(i) $\sigma(\triangle_{ab}, bv_0) = \bar{D} \cup E \cup F_0$,

(ii) $\sigma_p(\triangle_{ab}, bv_0) \subseteq E \cup K_0$

(iii) $\sigma_p(\triangle_{ab}^*, {bv_0}^*) = D \cup E \cup H_0$,

Where

$E = \{a_k : k \in \mathbb{N}, |a_k - a| > |b|\}$, **(2.1)**

$D = \{\lambda \in \mathbb{C} : |\lambda - a| < |b|\}$, **(2.2)**

$\bar{D} = \{\lambda \in \mathbb{C} : |\lambda - a| \le |b|\}$, **(2.3)**

$$F_0 = \{\lambda \in \mathbb{C} : |\lambda - a| > |b|;\ \sup_N \sum_{n=0}^{\infty} \left|\sum_{k=0}^{N}(d_{nk} - d_{n-1,k})\right| = \infty\}, \quad \textbf{(2.4)}$$

$$K_0 = \left\{a_j : j \in \mathbb{N}, |a_j - a| = |b|; \sum_{k=M_j}^{\infty}\left|\frac{b_k}{a_j - a_{k+1}} - 1\right|\left|\prod_{i=M_j}^{k-1}\frac{b_i}{a_j - a_{i+1}}\right| < \infty\right\}. \quad \textbf{(2.5)}$$

M_j is any fixed natural number such that $a_j - a_k \neq 0$ for all $k > M_j$, $j \in \mathbb{N}$ with $a_j \neq a$ and (a_k) is

a sequence of nonzero real numbers which converges to a.

$$G_0 = \{\lambda \in \mathbb{C} : |\lambda - a| > |b|, \sup_N \left|\sum_{k=0}^{N}\frac{(\lambda - a_0)(\lambda - a_1)\ldots(\lambda - a_k)}{b_0 b_1 \cdots b_k}\right| = \infty\} \quad \textbf{(2.6)}$$

$$H_0 = \{\lambda \in \mathbb{C} : |\lambda - a| = |b|, \sup_n \left|\sum_{k=0}^{n}\frac{(\lambda - a_0)(\lambda - a_1)\ldots(\lambda - a_k)}{b_0 b_1 \cdots b_k}\right| < \infty\}, \quad \textbf{(2.7)}$$

If $|a_j - a| \neq |b|$, for all $j \in \mathbb{N}$. so, $K_0 = \emptyset$. Then, we have the following results.

Theorem 2.2. The following statements hold:

(i) $\sigma_r(\triangle_{ab}\ , bv_0) = D \cup H_0$,

(ii) $\sigma_c(\triangle_{ab}\ , bv_0) = [\{\lambda \in \mathbb{C}: |\lambda - a| = |b|\} \backslash H_0] \cup F_0$.

Theorem 2.3. The following statements hold:

(i) $I_3(\triangle_{ab}\ , bv_0) = II_3(\triangle_{ab}\ , bv_0) = \emptyset$,

(ii) $III_3(\triangle_{ab}\ , bv_0) = E$,

(iii) $III_1(\triangle_{ab}\ , bv_0) \cup III_2(\triangle_{ab}\ , bv_0) = D \cup H_0$,

(iv) $I_2(\triangle_{ab}\ , bv_0) = \emptyset$, by the closed graph theorem.

(v) $II_2(\triangle_{ab}\ , bv_0) = [\{\lambda \in \mathbb{C}: |\lambda - a| = |b|\} \backslash H_0] \cup F_0$,

(vi) $\sigma_{ap}(\triangle_{ab}\ , bv_0) = (\overline{D} \setminus III_1(\triangle_{ab}, bv_0)) \cup E \cup F_0$,

(vii) $\sigma_{co}(\triangle_{ab}\ , bv_0) = D \cup E \cup H_0$,

(viii) $\sigma_\delta(\triangle_{ab}\ , bv_0) = \overline{D} \cup E \cup F_0$,

where E, F_0, K_0 and H_0 are given as in (2.1), (2.4), (2.5) and (2.7), respectively.

3. Main Results

Our work is an extension for the work in [4].

The generalized difference operator $\triangle^{ab}$ is defined on the Banach sequence space μ as follows

$$\triangle^{ab} x := (a_k x_k + b_k x_{k+1})_{k=0}^{\infty}, \; x = (x_k)_{k=0}^{\infty} \in \mu,$$

where (a_k) and (b_k) are given sequences, which are assumed to satisfy certain conditions

This operator can be represented by certain infinite upper triangular double-band matrix, as

$$\triangle^{ab} = \begin{bmatrix} a_0 & b_0 & 0 & 0 & \cdots \\ 0 & a_1 & b_1 & 0 & \cdots \\ 0 & 0 & a_2 & b_2 & \cdots \\ 0 & 0 & 0 & a_3 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \textbf{(3.1)}$$

Let (a_k) and (b_k) be two convergent sequences of nonzero real numbers and these sequences are such that

$$\lim_{k\to\infty} a_k = a,$$

$$\lim_{k\to\infty} b_k = b \neq 0.$$

So, under the assumption that the sequences (a_k) and (b_k) are bounded sequences of nonzero real numbers.

We consider the operator $\triangle^{ab}: bs \to \text{bs}$, which is defined by Eq. (3.1), for $\mu = bs$.

The operator $\triangle^{ab}$ which was represented by the matrix in (3.1) on bs is linear and bounded. It is easy to prove that.

The main aim of this paper to study the spectrum and fine spectrum of the generalized difference operator $\triangle^{ab}$ on the sequence space bs by using new techniques without many details about the proofs.

The work [4] related to lower triangular double-band matrices $\triangle_{ab}$ on the sequence space bv_0, the inverse of the matrix was calculated and used to determine the fine spectrum. Here we use a different simple method which helps us to determine the fine spectrum of $\triangle^{ab}$ on the sequence space bs without calculating the inverse of the double-band matrix by using the results in Theorems (2.1) and (2.2).

If $T: bs \to bs$ is a bounded linear operator with the matrix A, then the adjoint operator $T^*: bs^* \to bs^*$ is determined by the transpose A^t of the matrix A, where $bs^* \cong bv_0$.

Theorem 3.1.

$\sigma(\triangle^{ab}, bs) = \{\lambda \in \mathbb{C}: |\lambda - a| \leq |b|\} \cup E \cup F_0$.

where E and F_0 are given as in (2.1) and (2.4), respectively.

Proof. Since $bs^* \cong bv_0$. Then

$\sigma(\triangle^{ab}, bs) = \sigma(\triangle^{ab*}, bs^*)$, by (1.2) $= \sigma(\triangle_{ab}, bv_0) = \bar{D} \cup E \cup F_0 = \{\lambda \in \mathbb{C}: |\lambda - a| \leq |b|\} \cup E \cup F_0$.

Remark 3.1. Indeed, $\sigma(\triangle^{ab}, bs) \subseteq \bar{D} \cup E \cup G_0$, where E, $\bar{D}$ and G_0 is given as in (2.1), (2.3), (2.6).

Theorem 3.2.

(i) $\sigma_p(\triangle^{ab}, bs) = \{\lambda \in \mathbb{C}: |\lambda - a| < |b|\} \cup E \cup H_0$,

(ii) $\sigma_p(\triangle^{ab*}, bs^*) \subseteq E \cup K_0$,

where E, F_0, K_0 and H_0 are given as in (2.1),(2.4),(2.5) and (2.7), respectively.

Proof. $\sigma_p(\triangle^{ab}, bs) = \sigma_p(\triangle^*_{ab}, {bv_0}^*) = D \cup E \cup H_0 = \{\lambda \in \mathbb{C}: |\lambda - a| < |b|\} \cup E \cup H_0$.

$$\sigma_p(\triangle^{ab*}, bs^*) = \sigma_p(\triangle_{ab}, bv_0) \subseteq E \cup K_0.$$

If $|a_j - a| \neq |b|$, for all $j \in \mathbb{N}$. So, $K_0 = \emptyset$. Then, $\sigma_p(\triangle_{ab}, bv_0) = E$. So, **we obtain:**

Theorem 3.3. $\sigma_r(\triangle^{ab}, bs) = \emptyset$.

The proof follows from Theorem (3.2) and then applying Lemma 1.1.

$$\sigma_r(\triangle^{ab}, bs) = \sigma_p(\triangle^{ab*}, bs^*) \backslash \sigma_p(\triangle^{ab}, bs) = \emptyset.$$

Theorem 3.4.

$$\sigma_c(\triangle^{ab}, bs) = [\{\lambda \in \mathbb{C}: |\lambda - a| = |b|\} \setminus H_0] \cup F_0,$$

where F_0, H_0 and are given as in (2.4) and (2.7), respectively.

Proof.

$$\sigma_c(\triangle^{ab}, bs) = \sigma(\triangle^{ab}, bs) \backslash [\sigma_p(\triangle^{ab}, bs) \cup \sigma_r(\triangle^{ab}, bs)]$$

$$= [\{\lambda \in \mathbb{C}: |\lambda - a| = |b|\} \setminus H_0] \cup F_0.$$

Since,

$\sigma_p(\triangle^{ab}, bs) = I_3(\triangle^{ab}, bs) \cup II_3(\triangle^{ab}, bs) \cup III_3(\triangle^{ab}, bs)$,

$\sigma_r(\triangle^{ab}, bs) = III_1(\triangle^{ab}, bs) \cup III_2(\triangle^{ab}, bs)$,

$\sigma_c(\triangle^{ab}, bs) = II_2(\triangle^{ab}, bs)$,

$\sigma_{ap}(\triangle^{ab}, bs) = \sigma(\triangle^{ab}, bs) \setminus III_1(\triangle^{ab}, bs)$, by relation (1.1)

$\sigma_\delta(\triangle^{ab}, bs) = \sigma(\triangle^{ab}, bs) \setminus I_3(\triangle^{ab}, bs)$, by relation (1.1)

$\sigma_{co}(\triangle^{ab}, bs) = \sigma_p(\triangle^{ab*}, bs^*)$. by relation (1.3)

Thus, we have this theorem.

Theorem 3.5. If $\left|a_j - a\right| \neq |b|$, for all $j \in \mathbb{N}$**.** So**,** $K_0 = \emptyset$**.** Then.

(i) $I_3(\triangle^{ab}, bs) \cup II_3(\triangle^{ab}, bs) = \sigma_p(\triangle^{ab}, bs) \setminus \sigma_p(\triangle^{ab*}, bs^*)$
$= \{\lambda \in \mathbb{C}: |\lambda - a| < |b|\} \cup H_0$,
(ii) $III_3(\triangle^{ab}, bs) = \mathrm{E}$,
(iii)$III_1(\triangle^{ab}, bs) \cup III_2(\triangle^{ab}, bs) = \emptyset$,
(iv)$II_2(\triangle^{ab}, bs) = [\{\lambda \in \mathbb{C}: |\lambda - a| = |b|\} \setminus H_0] \cup F_0$,
(v) $\sigma_{ap}(\triangle^{ab}, bs) = \{\lambda \in \mathbb{C}: |\lambda - a| \leq |b|\} \cup E \cup F_0$,
(vi)$\sigma_\delta(\triangle^{ab}, bs) = \{\lambda \in \mathbb{C}: |\lambda - a| \leq |b|\} \cup E \cup F_0 \backslash I_3(\triangle^{ab}, bs)$,
(vii) $\sigma_{co}(\triangle^{ab}, bs) = E$.

where E,F_0 and H_0 are given as in (2.1),(2.4) and (2.7), respectively.

Conclusion

Many results were obtained with respect to the fine spectrum of upper triangular double-band matrices as operators on certain sequence spaces. But, no contribution has appeared so far to study the problem in the sequence space bs. In this paper, we fill this gap.

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