On Solving Linear Programming Problem with Fuzzy Coefficients

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Abstract:

This paper deals with linear programming problem through fuzzy numbers coefficients FLPP. The FLPP with uncertainty parameter which is embodiment in Triangular fuzzy number in both the objective function and constraint, then convert it into the crisp liner programming problem CLPP. A proposed approach of solution assists me to achieve the interval optimal solution of FLPP as clarify via numerical example in this paper.

Key words : Linear programming Problem, Fuzzy numbers, α-cut, Optimization problem.

الملخص

يهتم هذا البحث بدراسة مشكلة البرمجة الخطية ذات المعاملات (الغامضة) الفازية, وتحويل هده المشكلة إلي مشكلة يقينية و تجزئتها إلى مشكلتين يقينيتين وحل كل مشكلة علي حدا ثم دمج حلولهما، ونوضح خطوات حل هذه المشكلة بمثال عددي.

1. Introduction

Fuzzy linear programming problem (FLPP) is useful in solving problems which are difficult, impossible to solve due to the imprecise, subjective natureof the problem formulation or have an accurate solution. In this paper i discussed the concepts of fuzzy decision making as mentioned by[1][2] and the maximum decision [11] that is used in LPP to find the optimal decision(solution). This decision making used in fuzzy linear programming problem[8] and [9]. this problem has fuzzy objective function and fuzzy variables in the constraints [6] and [12] where the lower interval and upper interval coefficients on objective and constraints [10]. we introduce some definitions that are useful in our problem. we state linear programming in fuzzy environment by transform the crisp problem. Finally, we illustrated by numerical example and the conclusions.

Preliminaries

We introduce some notions related to the FLLP. we have some concepts can be found in [3],[4] [5] and [7]. Number of α-cuts, leading to a series of dual intervals being generated. Among these intervals, the internal has two limits $a^{L}(\alpha)$ and $a^{U}(\alpha)$).

Definition: Here we define a triangular fuzzy numbers represented in the form of interval of confidence level $\alpha \in [0,1]$. The membership function for the triangular fuzzy number $A = (a_1, a_2, a_3)$ is defined as :

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$$\mu_a(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0, & x \ge a_3. \end{cases}$$

The interval of confidence for the triangular fuzzy number A = (a_1, a_2, a_3) at the level α, is defined as:
$A_\alpha = [a_{1\alpha},\ a_{2\alpha}] = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3] \quad \forall\alpha \in [0,1]$

Definition: Let $F_N(R)$ denote the set of all compact fuzzy numbers, that is, for any

$\tilde{a} \in F_N(R)$, satisfies:

1. $\tilde{a}$is normal $(i.e \exists x \in a_\alpha = \{x : \tilde{a}(x) \ge \alpha, \alpha \in (0,1]\}$)such that$\tilde{a}(x) = 1$,
2. For any $\alpha \in (0,1]$, $a_\alpha = [a^L(\alpha), a^U(\alpha)]$ is closed interval number on *R*, $a^L_\alpha \le a^U_\alpha$.

Suppose that, $a_\alpha = [\, a^L(\alpha),\ a^U(\alpha)\,]$, $b_\alpha = [\, b^L(\alpha),\ b^U(\alpha)\,]$, we define

1. $a_\alpha + b_\alpha =$

$$[a^L(\alpha), a^U(\alpha)] + [b^L(\alpha), b^U(\alpha)] = [a^L(\alpha) + b^L(\alpha), a^U(\alpha) + b^U(\alpha)], \quad (1)$$

2. $a_\alpha - b_\alpha$=

$$[\mathrm{a^L}(\alpha), \mathrm{a^U}(\alpha)] - [\mathrm{b^L}(\alpha), \mathrm{b^U}(\alpha)] = [\mathrm{a^L}(\alpha) - \mathrm{b^U}(\alpha), \mathrm{a^U}(\alpha) - \mathrm{b^L}(\alpha)], \quad (2)$$

3. $a_\alpha \,.\, b_\alpha = [a^L(\alpha), a^U(\alpha)] \,.\, [b^L\alpha, b^U(\alpha)]$

$$= [a^L(\alpha).b^L(\alpha) \wedge a^U(\alpha).b^L(\alpha) \wedge a^L(\alpha).b^U(\alpha) \wedge a^U(\alpha).b^U(\alpha),\ a^L(\alpha).b^L(\alpha) \vee a^L(\alpha).b^U(\alpha) \vee a^U(\alpha).b^L(\alpha) \vee a^U(\alpha).b^U(\alpha)]$$

(3)

Problem formulations:

Consider the fuzzy linear programming problem (FLPP), as in the following form:

FLPP $\min_{\tilde{x} \in \tilde{M}} \left(f(\tilde{C}, \tilde{X}) = \tilde{C}\tilde{X}\right)$ (4)

Subject to:

$$\tilde{M}(\tilde{A}, \tilde{B}) = \{\tilde{x} : \tilde{A}\tilde{X} \le \tilde{B}\ , \quad \tilde{x} \ge 0\}$$

Where$\tilde{C} = [C_j]_{1\times n}, \tilde{A} = [\tilde{a}_{ij}]_{m\times n}$, $\tilde{B} = (\tilde{b}_1, \tilde{b}_2, \dots, \tilde{b}_m)$, $X = (x_1, x_2, \dots, x_n)^T$,and $\tilde{D}, \tilde{A}, \tilde{B}$ are compact fuzzy numbers.

Definition :

The fuzzy vector $x^*(\tilde{A}^*, \tilde{B}^*)$ which satisfies the conditions of FLPP

, is said to be fuzzy optimal solution of FLPP if

$$\tilde{f}(\tilde{C}^*, \tilde{x}^*) \le \tilde{f}(\tilde{C}^*, \tilde{x})$$

For each

$\tilde{x}^* \in \widetilde{M}(\tilde{A}, \tilde{B})$ furthermore if $\tilde{x}^*$ is a fuzzy vector then it is

,said to be fuzzy optimal solution of FLPP:

For any $\alpha \in (0, 1]$ the α-level cuts of $c_i^*, a_{ji}^*, b_j^*, x_i^*$ are intervals denote them by:

$$(c_i^*)_\alpha = [c_i^{*L}(\alpha), c_i^{*U}(\alpha)]$$

$$(a_{ji}^*)_\alpha = [a_{ji}^{*L}(\alpha), a_{ji}^{*U}(\alpha)],$$

$$(b_j^*)_\alpha = [b_j^{*L}(\alpha), b_j^{*U}(\alpha)],$$

$$(x_i^*)_\alpha = [x_i^{*L}(\alpha), x_i^{*U}(\alpha)],$$

i= 1,2,…, n j = 1, 2,…, m

For any $\alpha \in (0, 1]$ we convert the problem to be two problems upper $\tilde{P}_U$ and lower $\tilde{P}_L$ as follows:

$P_U: \min_{x\in M^U} (f^U = C^U X^U)$

Subject to:

$$M^U = \{x : A^U X \le B^U, x \ge 0\}$$

$P_L: \min_{x\in M^L} (f^L = C^L X^L)$

Subject to:

$$M^L = \{x : A^L X \le B^L, x \ge 0\}$$

The Proposed Approach

The proposed approach built based on the concept of:

1. Convert FLPP into crisp linear programming problem CLPP with use the $\alpha - cut$ of all the fuzzy numbers coefficients in problem.
2. CLPP divided to upper problem P_U and lower problem P_L.
3. Solving the upper problem P_U to get the solution x^{*U} .
4. Add constraint $x^L \leq x^{*U}$ to the constraint of the lower problem P_L then solve it to get this solution x^{*L} of P_L.
5. Incorporate the solution of lower problem and upper problem, to achieve the optimal solution as following:

$$x_\alpha = [x_\alpha^{*L}, x_\alpha^{*U}]$$

Example: Consider the following fuzzy linear programming problem(FLPP):

$$Max(\tilde{2} \quad \tilde{3})\begin{pmatrix}\tilde{x}_1\\ \tilde{x}_2\end{pmatrix}$$

Subject to:

$$\begin{pmatrix}\tilde{3} & \tilde{5}\\ \tilde{8} & \tilde{7}\\ \tilde{3} & \tilde{4}\end{pmatrix}\begin{pmatrix}\tilde{x}_1\\ \tilde{x}_2\end{pmatrix} \leq \begin{pmatrix}\widetilde{30}\\ \widetilde{50}\\ \widetilde{28}\end{pmatrix}, \qquad \begin{pmatrix}\tilde{x}_1\\ \tilde{x}_2\end{pmatrix} \geq 0$$

We take α= 0.5 then, CLPP as:

$$Max([1.5,2.5] \quad [2.5,3.5])\begin{pmatrix}[x_1^L, x_1^U]\\ [x_2^L, x_2^U]\end{pmatrix}$$

Subject to:

$$\begin{pmatrix}[2.5,3.5] & [4.5, 5.5]\\ [7.5,8.5] & [6.5, 7.5]\\ [2.5,3.5] & [3.5, 4.5]\end{pmatrix}\begin{pmatrix}[x_1^L, x_1^U]\\ [x_2^L, x_2^U]\end{pmatrix} \leq \begin{pmatrix}[29.5,30.5]\\ [49.5,50.5]\\ [27.5,28.5]\end{pmatrix}$$

Now, we divided a problem above to a problems are upper problem $P^U(\alpha)$ and a lower problem $P^L(\alpha)$ follows

$P^L(\alpha)$:

$$Max(\, 1.5 \quad 2.5)\begin{pmatrix}x_1^u\\ x_2^u\end{pmatrix}$$

Subject to:

$$\begin{pmatrix}2.5 & 4.5\\ 7.5 & 6.5\\ 2.5 & 3.5\end{pmatrix}\begin{pmatrix}[x_1^L]\\ [x_2^L]\end{pmatrix} \leq \begin{pmatrix}29.5\\ 49.5\\ 27.5\end{pmatrix}$$

$P^U(\alpha)$

$$Max(\,2.5 \quad 3.5)\begin{pmatrix} x_1^U \\ x_2^U \end{pmatrix}$$

Subject to:

$$\begin{pmatrix} 3.5 & 5.5 \\ 8.5 & 7.5 \\ 3.5 & 4.5 \end{pmatrix}\begin{pmatrix} x_1^U \\ x_2^U \end{pmatrix} \leq \begin{pmatrix} 30.5 \\ 50.5 \\ 28.5 \end{pmatrix}$$

The solution of P^U is $x_1^{*U} = 2.39$, $x_2^{*U} = 4.02$ and the optimal solution is $f^{*U} = 20.06$

The solution of P^L is $x_1^{*L} = 1.77$, x_2^{*L} =5.57 and the optimal solution is $x^u = f^{*L} = 16.58$.

Follows the solution of problem CLPP is

$$x^* = \ (x_1^{*L}, x_2^{*U}) = (\,[1.77, 2.39], \qquad [4.02, 5.57])$$

The optimal solution of fuzzy linear programming problem is $f^* = \left[f^{*^L}, f^{*^U}\right] = [16.58\,, 20{,}06]$

Conclusion

In this paper, we have obtained an optimal solution of linear programming problem with fuzzy coefficients in both objective and constraints. The FLPP is transformed into crisp linear programming problem. The new arithmetic operations based convert crisp problem to two problems corresponding to the upper and lower interval, respectilvely. Solve all the problems and incorporate the solution of all problems to obtain optimal solution as follows interval.

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