Constructing Exact Solutions of the Burgers-Huxley Equation via (G'/G) - Expansion Method

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Abstract

This study derives exact analytical solutions to the Burgers–Huxley equation by employing the extended (G'/G)-expansion method. By applying the traveling wave transformation: $\eta = x - ct$, the nonlinear partial differential equation is reduced to an ordinary differential equation. The method is based on a finite expansion in terms of (G'/G), where the order of expansion is determined using the homogeneous balance principle. Substitution into the reduced equation generates a system of algebraic equations for the unknown parameters, whose solutions yield several classes of exact traveling wave solutions. These solutions are expressed in hyperbolic, trigonometric, and rational functional forms, and can be categorized into distinct nonlinear wave structures, including solitary waves and kink-type solutions. The kink-type solutions characterize transition waves connecting two different asymptotic states, whereas the solitary wave solutions represent localized, stable structures. The obtained results highlight the effectiveness of the Extended -expansion method in constructing diverse families of exact solutions for nonlinear evolution equations.

Keywords: *Burgers–Huxley equation, extended (G'/G) expansion method, kink solutions, solitary waves, exact solutions.*

الملخص

تستخلص هذه الدراسة حلولًا تحليلية دقيقة لمعادلة Burgers–Huxley باستخدام طريقة "extended (G'/G)-expansion". ومن خلال تطبيق تحويل الموجة المسافرة، يتم اختزال المعادلة التفاضلية الجزئية غير الخطية إلى معادلة تفاضلية عادية. وتعتمد الطريقة على توسع محدود بدلالة (G'/G)، حيث يتم تحديد رتبة التوسع باستخدام مبدأ التوازن المتجانس. ويؤدي التعويض في المعادلة المختزلة إلى نظام من المعادلات الجبرية للمعاملات المجهولة، وتوفر حلولها عدة أصناف من حلول الموجات المسافرة الدقيقة. تُعبر هذه الحلول في صيغ دوال زائدية، مثلثية، وقياسية، ويمكن تصنيفها ضمن هياكل موجية غير خطية مميزة، بما في ذلك الموجات الانفرادية وحلول من نوع "kink". تمثل حلول الـ "kink" موجات انتقالية تربط بين حالتين حدّيتين مختلفتين، في حين تمثل الحلول الانفرادية تراكيب موضعية مستقرة. وتبرز النتائج فعالية طريقة "extended (G'/G)-expansion" في بناء عائلات متنوعة من الحلول الدقيقة لمعادلات التطور غير الخطية.

الكلمات المفتاحية: *معادلة Burgers–Huxley، طريقة "extended (G'/G)-expansion"، حلول "kink" موجات انفرادية، حلول دقيقة.*

1-Introduction

Partial Differential Equations (PDEs) of the nonlinear type constitute one of the most powerful mathematical tools for modeling a wide range of physical, biological, and chemical phenomena. They provide precise descriptions of the complex interplay among diffusion, convection, and reaction processes in natural systems. Over the past few decades, nonlinear equations have attracted considerable attention from researchers, leading to the development of numerous analytical techniques for obtaining exact solutions. such as the first integral method (Abdoon, 2015), the exp-function method (He, 2013) and (El-borai et al., 2015), the functional variable method(Zerarka et al., 2010) and (Proceedings et al., 2016), the Jacobi elliptic function method,(Maruno et al., 2003), (Gepreel, 2014) and (Maruno et al., 2003), the tanh-function method,(A. Wazwaz, 2007), (Parkes, 2009) and (El-borai & Al-masroub, 2015), the(G′/G)-Expansion method], (El-borai & Al-masroub, 2015) and (Zheng, 2013), the Kudryashov method,(Kumar & Chand, 2014) and (Zayed & Alurrfi, 2015), (G′/G,1/G)-Expansion method,(Zayed & Alurrfi, 2014) and (Al-shawba et al., 2018),the generalized Riccati equation mapping method,(Li et al., 2014) and (El-borai et al., 2016) and others.

Among these nonlinear equations, the Burgers–Huxley equation stands out as an important mathematical model that combines the features of both the Burgers equation and the Huxley equation. This combination enables it to represent a broad spectrum of phenomena, including nonlinear wave propagation, fluid dynamics, and pulse transmission in reactive systems. The Burgers equation was originally introduced by the Dutch physicist Johannes Martinus Burgers in the context of turbulence and momentum transfer in fluids, whereas Huxley-type models are associated with reaction–diffusion processes in biological systems. Merging these two frameworks into a single equation provides a rich mathematical setting for studying nonlinear wave propagation in diverse media. Several methods have been employed to derive exact solutions of the Burgers–Huxley equation in various forms. In (Maurya et al., 2019), the New Homotopy Perturbation Method was applied, leading to the derivation of exact solitary wave solutions. In (Ebiwareme, 2021) and (A. M. Wazwaz, 2019), the Tanh–coth method was utilized, yielding both solitary and traveling wave solutions.

The primary objective of this research is to apply the extended (G′/G)-expansion method to the Burgers–Huxley equation in order to obtain solitary wave and exact analytical solutions, thereby demonstrating the method's capability and efficiency in addressing highly nonlinear partial differential equations arising in physics, mathematics, and engineering. In addition, a comparison is made between the proposed method and the classical tanh–coth method reported in References (Ebiwareme, 2021) and (A. M. Wazwaz, 2019), in order to highlight the advantages and generalization achieved by the extended (G′/G)-expansion method expansion framework. The extended -expansion method, proposed by Erdal Fan (Fan, 2000), has proven highly effective in deriving exact wave solutions for a wide class of nonlinear differential equations. This method is based on the assumption that the solution can be expressed as a series expansion in terms of the ratio, where the function satisfies a second-order linear ordinary differential equation.

The structure of this paper is organized as follows: Section 2 presents the theoretical foundation of the extended (G'/G)-expansion method. Section 3 discusses the application of this method to the Burgers-Huxley equation. Section 4 provides graphical representations of the proposed solutions. Section 5 concludes the work with a brief summary. Finally, Section 6 lists the references used throughout the paper.

2- Description of the Extended (G'/G) -Expansion method Consider the partial differential equation:

$$Q(v, v_t, v_x, v_{tt}, v_{xx}, \dots) = 0, \tag{1}$$

Where $v = v(x,t)$an unknown is function, and Q is a polynomial in$v(x,t)$.

The main steps of the extended(G'/G)-expansion method, together with associated Riccati equation$(G'' + \lambda G' + \mu G = 0)$ are formulated as follows:(El-borai & Al-masroub, 2015) and (Zheng, 2013)

Step 1. Consider the wave transform:

$$v(x,t) = v(\eta), \text{where } \eta = x - ct, \tag{2}$$

Where c is a nonzero constant and $v(\eta)$ is a function, of η.Substituting Equation (2) into Equation (1) yields an ordinary differential equation for $v(\eta)$ of the form:

$$R\left(v, v_\eta, v_\eta, v_{\eta\eta}, v_{\eta\eta}, \dots\right) = 0 \tag{3}$$

Step 2. Suppose that the solution of Equation (3), can be expressed as a polynomial in(G'/G) as follows:

$$v(\eta) = \sum_{i=-1}^{M} a_i (G'/G)^i = 0 \tag{4}$$

Wherea_i are constants (i=1, 2, 3…M) to be determined later, and $g = g(\eta)$satisfies the Riccati equation:

$$G''(\eta) + \lambda G'(\eta) + \mu G(\eta) = 0. \tag{5}$$

Step 3. The positive integer "M" can be determined by considering the homogeneous balance between the highest derivative term and the nonlinear terms appearing in Equation. (3). Suppose that:

$$\varphi(\eta) = \frac{G'(\eta)}{G(\eta)}. \tag{6}$$

Substitute equation (6) into equation (4). Then equation (4) becomes

$$v(\eta) = \sum_{i=-1}^{M} a_i \varphi^i(\eta) = 0. \tag{7}$$

Differentiate equation (6) and using equation ((5), we obtain

$$\begin{cases} \varphi'(\eta) = -\lambda\varphi(\eta) - \varphi^2(\eta) - \mu \\ \varphi''(\eta) = 2\varphi^3(\eta) + 3\lambda\varphi^2(\eta) + (\lambda^2 + 2\mu)\varphi(\eta) + \lambda\mu \\ \varphi''(\eta) = -6\varphi^4(\eta) - 12\varphi^3(\eta)\lambda + (-7\lambda^2 - 8\mu)\varphi^2 2(\eta) + (-\lambda^3 - 8\lambda\mu)\varphi(\eta) - \lambda^2\mu \\ \quad\vdots \end{cases} \tag{8}$$

Substitute equation (7) into equation (3) with the help of equation (8).By collecting all terms with the same powers of $\varphi^i(\eta)$and equating their coefficients to zero, we obtain a system of algebraic equations for: $a_{-1}, a_0, a_1, \lambda$, c and k.Solving this system using Maple yields different sets of solutions. Substituting these sets into equation (7) and employing the known solutions of equation (5), namely

$$\varphi(\eta)=\begin{cases}\frac{\sqrt{\Omega}}{2}\left(\frac{\upsilon_1\sinh\left(\frac{\sqrt{\Omega}}{2}\eta\right)+\upsilon_2\cosh\left(\frac{\sqrt{\Omega\mu}}{2}\eta\right)}{\upsilon_1\cosh\left(\frac{\sqrt{\Omega}}{2}\eta\right)+\upsilon_2\sinh\left(\frac{\sqrt{\Omega}}{2}\eta\right)}\right)-\frac{\lambda^2}{2},\Omega>0\\ \frac{\sqrt{\Omega}}{2}\left(\frac{\upsilon_1\sin\left(\frac{\sqrt{\Omega}}{2}\eta\right)+\upsilon_2\cos\left(\frac{\sqrt{\Omega}}{2}\eta\right)}{\upsilon_1\cos\left(\frac{\sqrt{\Omega}}{2}\eta\right)+\upsilon_2\sin\left(\frac{\sqrt{\Omega}}{2}\eta\right)}\right)-\frac{\lambda^2}{2},\qquad \Omega<0\\ \frac{\upsilon_2}{\upsilon_1+\eta\upsilon_2}-\frac{\lambda^2}{2},\Omega=0,\end{cases}\tag{9}$$

Where$\Omega=\sqrt{\lambda^2-4\mu}$, υ_1 and υ_2 are arbitrary constants. We have the traveling wave solutions of the nonlinear partial differential equation (1).

3- **Application of the extended (G′/G) expansion method of Burgers-Huxley equation given the Burger-Huxley equation as:**

$$(c+v)\frac{\partial v}{\partial\eta}+\frac{\partial^2 v}{\partial\eta^2}+v(k-v)(v-1)=0.\tag{10}$$

Suppose that the solution of ODEs (10) can be expressed as a polynomial in the form:

$$v(\eta)=\sum\nolimits_{i=-1}^{M}a_i(G'/G)^i=0\tag{11}$$

Where a_i are constants (i=1, 2, 3…M) to be determined later, and $g=g(\eta)$satisfies the Riccati equation:

$$G''+\lambda G'+\mu G=0.\tag{12}$$

Considering the homogeneous balance between the highest order derivatives (v'') and the nonlinear terms (v^3) in Equation. (10). Thus, we have

$$3M=M+2\Longrightarrow M=2$$

Substitute into equation (11), we obtain:

$$v(\eta) = \frac{a_{-1}}{\varphi(\eta)} + a_0 + a_1\,\varphi(\eta) \tag{13}$$

Substitute equation (13) into equation (10) with the help of equation (8). By collecting all terms with the same powers of $\varphi^i(\eta)$and equating their coefficients to zero, we obtain the following system of algebraic equations for the parameters: $a_{-1}, a_0, a_1, \lambda$, c,and k:

$$\varphi(\xi)^{-3} : -2\mu^2 a_{-1} - \mu a_{-1}^2 + a_{-1}^3 = 0,$$

$$\varphi(\xi)^{-2} : -c\mu a_{-1} - k a_{-1}^2 - 3\lambda\mu a_{-1} - \lambda a_{-1}^2 - \mu a_{-1} a_0 + 3 a_{-1}^2 a_0 - a_{-1}^2 = 0,$$

$$\varphi^{-1}(\xi) : -c\lambda a_{-1} - 2k a_{-1} a_0 - \lambda^2 a_{-1} - \lambda a_{-1} a_0 + 3 a_{-1}^2 a_1 + 3 a_{-1} a_0^2 + k a_{-1} - 2\mu a_{-1} - a_{-1}^2 - 2 a_{-1} a_0 = 0,$$

$$\varphi^0(\xi) : c a_1 \mu - 2 a_{-1} k a_1 - a_0^2 k - a_1 \lambda \mu + a_0 a_1 \mu + 6 a_{-1} a_0 a_1 + a_0^3 - c a_{-1} + a_0 k - a_{-1}\lambda - a_0 a_{-1} - 2 a_{-1} a_1 - a_0^2 = 0,$$

$$\varphi(\xi) : c\lambda a_1 - 2k a_0 a_1 - \lambda^2 a_1 + \lambda a_0 a_1 + \mu a_1^2 + 3 a_{-1} a_1^2 + 3 a_0^2 a_1 + k a_1 - 2\mu a_1 - 2 a_0 a_1 = 0,$$

$$\varphi(\xi)^2 : -k a_1^2 + \lambda a_1^2 + 3 a_0 a_1^2 + c a_1 - 3\lambda a_1 + a_0 a_1 - a_1^2 = 0,$$

$$\varphi(\xi)^3 : a_1^3 + a_1^2 - 2 a_1 = 0.$$

By solving this system using Maple, we obtain the following three sets of solutions**:**

Case 1. For $a_{-1} = 0$, we have:

$$S_1 = \left\{c = k - 1, \mu = \frac{1}{4}\lambda^2 - \frac{1}{4}, a_{-1} = 0, a_0 = \frac{1}{2}\lambda + \frac{1}{2}, a_1 = 1\right\},$$

$$S_2 = \left\{c = -k + 1, \mu = -\frac{1}{4}k^2 + \frac{1}{4}\lambda^2, a_{-1} = 0, a_0 = \frac{1}{2}\lambda + \frac{1}{2}k, a_1 = 1\right\},$$

$$S_3 = \left\{c = -k - 1, \mu = \frac{1}{2}k - \frac{1}{4}k^2 + \frac{1}{4}\lambda^2 - \frac{1}{4}, a_{-1} = 0, a_0 = \frac{1}{2} + \frac{1}{2}\lambda + \frac{1}{2}k, a_1 = 1\right\},$$

$$S_4 - \left\{c = \frac{1}{2}k - 2, \mu = -\frac{1}{16}k^2 + \frac{1}{4}\lambda^2, a_{-1} = 0, a_0 = -\lambda + \frac{1}{2}k, a_1 = -2\right\},$$

$$S_5=\left\{c=\frac{1}{2}k+\frac{1}{2},\mu=\frac{1}{8}k-\frac{1}{16}k^2+\frac{1}{4}\lambda^2-\frac{1}{16},a_{-1}=0,a_0=-\lambda+\frac{1}{2}+\frac{1}{2}k,a_1=-2\right\},$$

$$S_6=\left\{c=-2k+\frac{1}{2},\mu=\frac{1}{4}\lambda^2-\frac{1}{16},a_{-1}=0,a_0=-\lambda+\frac{1}{2},a_1=-2\right\}.$$

Case 2. For $a_1=0$, we have:

$$S_7=\left\{c=k-1,\mu=\frac{1}{4}\lambda^2-\frac{1}{4},a_{-1}=-\frac{1}{4}\lambda^2+\frac{1}{4},a_0=-\frac{1}{2}\lambda+\frac{1}{2},a_1=0\right\},$$

$$s_8=\left\{c=-1-k,\mu=-\frac{1}{4}(-1-k)^2+\frac{1}{4}\lambda^2+k,a_{-1}=\frac{1}{4}(-1-k)^2-\frac{1}{4}\lambda^2-k,\right.$$
$$\left.a_0=-\frac{1}{2}\lambda+\frac{1}{2}+\frac{1}{2}k,a_1=0\right\}$$

$$s_9=\left\{c=-k+1,\mu=-\frac{1}{2}k+\frac{1}{4}-\frac{1}{4}(-k+1)^2+\frac{1}{4}\lambda^2,a_{-1}\right.$$
$$=\frac{1}{2}k-\frac{1}{4}+\frac{1}{4}(-k+1)^2-\frac{1}{4}\lambda^2,$$
$$\left.a_0=-\frac{1}{2}\lambda+\frac{1}{2}k,a_1=0\right\}$$

$$S_{10}=\left\{c=\frac{1}{2}-2k,\mu=\frac{1}{4}\lambda^2-\frac{1}{16},a_{-1}=\frac{1}{2}\lambda^2-\frac{1}{8},a_0=\lambda+\frac{1}{2},a_1=0\right\},$$

$$s_{11}=\left\{c=\frac{1}{2}+\frac{1}{2}k,\mu=\frac{1}{8}k-\frac{1}{16}-\frac{1}{16}k^2+\frac{1}{4}\lambda^2,a_{-1}=-\frac{1}{8}+\frac{1}{4}k-\frac{1}{8}k^2+\frac{1}{2}\lambda^2,\right.$$
$$\left.a_0=\lambda+\frac{1}{2}+\frac{1}{2}k,a_1=0\right\}$$

$$S_{12}=\left\{c=\frac{1}{2}k-2,\mu=-\frac{1}{16}k^2+\frac{1}{4}\lambda^2,a_{-1}=-\frac{1}{8}k^2+\frac{1}{2}\lambda^2,a_0=\frac{1}{2}k+\lambda,a_1=0\right\}.$$

Case3.For $a_{-1}$and, $a_1$which are not equal to zero, we have:

$$S_{13}=\{c=-6,k=7,\lambda=-3,\mu=2,a_{-1}=-2,a_0=5,a_1=-2\},$$

$$S_{14}=\left\{c=-\frac{1}{6},k=\frac{7}{6},\lambda=\frac{1}{2},\mu=\frac{1}{18},a_{-1}=-\frac{1}{18},a_0=\frac{1}{3},a_1=-2\right\},$$

$$S_{15}=\left\{c=-\frac{5}{6},k=-\frac{1}{6},\lambda=-\frac{1}{2},\mu=\frac{1}{18},a_{-1}=-\frac{1}{18},a_0=\frac{2}{3},a_1=-2\right\},$$

$S_{16}=\{c=5, k=-6, \lambda=3, \mu=2, a_{-1}=-2, a_0=-4, a_1=-2\}$,

$S_{17}=\left\{c=-\frac{6}{7}, k=\frac{1}{7}, \lambda=\frac{3}{7}, \mu=\frac{2}{49}, a_{-1}=\frac{4}{49}, a_0=\frac{5}{7}, a_1=1\right\}$,

$S_{18}=\{\{c=-6, k=7, \lambda=3, \mu=2, a_{-1}=4, a_0=5, a_1=1\}\}$,

$S_{19}=\left\{c=-\frac{1}{6}, k=\frac{7}{6}, \lambda=-\frac{1}{2}, \mu=\frac{1}{18}, a_{-1}=\frac{1}{9}, a_0=\frac{1}{3}, a_1=1\right\}$,

$S_{20}=\left\{c=-\frac{5}{6}, k=-\frac{1}{6}, \lambda=\frac{1}{2}, \mu=\frac{1}{18}, a_{-1}=\frac{1}{9}, a_0=\frac{2}{3}, a_1=1\right\}$,

$S_{21}=\{c=5, k=-6, \lambda=-3, \mu=2, a_{-1}=4, a_0=-4, a_1=1\}$,

$S_{22}=\left\{c=-\frac{1}{7}, k=\frac{6}{7}, \lambda=-\frac{3}{7}, \mu=\frac{2}{49}, a_{-1}=\frac{4}{49}, a_0=\frac{2}{7}, a_1=1\right\}$.

For (S_1), since $\lambda^2-4\mu=1>0$, so we have:

$$v_1(x,t)=\frac{1}{2}\left(1-\frac{\upsilon_1\sinh(\theta)-\upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta)-\upsilon_2\sinh(\theta)}\right),$$

By setting $\upsilon_2=0$and $\upsilon_1=0$, we respectively obtain the following solutions:

$$v_{1,1}(x,t)=\frac{1}{2}(1-\tanh(\theta)),$$

$$v_{1,2}(x,t)=\frac{1}{2}(1-\coth(\theta)),$$

Where, $\theta=\frac{1}{2}(k-1)t-\frac{1}{2}x$.

For (S_2), since $\lambda^2-4\mu=k^2>0$, so we have:

$$v_2(x,t)=\frac{1}{2}k\left(1+\frac{\upsilon_1\sinh(\theta)+\upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta)+\upsilon_2\sinh(\theta)}\right),$$

By setting $\upsilon_2=0$ and $\upsilon_1=0$, we respectively obtain the following solutions:

$$v_{2,1}(x,t)=\frac{1}{2}k(1+\tanh(\theta)),$$

$$v_{2,1}(x,t)=\frac{1}{2}k(1+\coth(\theta)),$$

Where, $\theta=\left(\frac{1}{2}k^2-\frac{1}{2}k\right)t+\frac{1}{2}kx$.

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For (S_3), since $\lambda^2 - 4\mu = (k-1)^2 > 0$, we have:

$$v_3(x,t) = \frac{1}{2}\left((k+1) + ((k-1)) \frac{(\upsilon_1 \sinh(\theta) + \upsilon_2 \cosh(\theta))}{\upsilon_1 \cosh(\theta) + \upsilon_2 \sinh(\theta)} \right),$$

By setting $\upsilon_2 = 0$ and $\upsilon_1 = 0$, we respectively obtain the following solutions:

$$v_{3,1}(x,t) = \frac{1}{2}((k+1) + ((k-1)) (\tanh(\theta))),$$

$$v_{3,1}(x,t) = \frac{1}{2}((k+1) + ((k-1)) (\coth(\theta))),$$

Where, $\theta = \left(\frac{1}{2}k^2 - \frac{1}{2}\right)t + \left(\frac{1}{2}k - \frac{1}{2}\right)x.$

For (S_4), since $\lambda^2 - 4\mu = (k-1)^2 > 0$, we have:

$$v_4(x,t)(x,t) = \frac{1}{2}k\left(1 - \frac{(\upsilon_1 \sinh(\theta) + \upsilon_2 \cosh(\theta))}{\upsilon_1 \cosh(\theta) + \upsilon_2 \sinh(\theta)} \right),$$

By setting $\upsilon_2 = 0$and $\upsilon_1 = 0$, we respectively obtain the following solutions:

$$v_{4,1}(x,t)(x,t) = \frac{1}{2}k(1 - (\tanh(\theta))),$$

$$v_{4,2}(x,t)(x,t) = \frac{1}{2}k(1 - (\coth(\theta))),$$

Where, $\theta = \left(-\frac{1}{8}k^2 + \frac{1}{2}k\right)t + \frac{1}{4}k\,x.$

For (S_5), since $\lambda^2 - 4\mu = \frac{(k-1)^2}{4} > 0$, we have:

$$v_5(x,t) = \frac{1}{2}\left((k+1) + (k-1) \frac{(\upsilon_1 \sinh(\theta) - \upsilon_2 \cosh(\theta))}{\upsilon_1 \cosh(\theta) - \upsilon_2 \sinh(\theta)} \right),$$

By setting $\upsilon_2 = 0$ and $\upsilon_1 = 0$, we respectively obtain the following solutions:

$$v_{5,1}(x,t) = \frac{1}{2}((k+1) + (k-1) (\tanh(\theta))),$$

$$v_{5,2}(x,t) = \frac{1}{2}((k+1) - (k-1) (\coth(\theta))),$$

Where $\theta = \left(\frac{1}{8}k^2 - \frac{1}{8}\right)t + \left(-\frac{1}{4}k + \frac{1}{4}\right)x.$

For (S_6), since $\lambda^2 - 4\mu = \frac{1}{4} > 0$ so, we have:

$$v_6(x,t) = \frac{1}{2}\left(1 - \frac{\upsilon_1 \sinh(\theta) + \upsilon_2 \cosh(\theta)}{\upsilon_1 \cosh(\theta) + \upsilon_2 \sinh(\theta)}\right),$$

By setting $\upsilon_2 = 0$ and $\upsilon_1 = 0$, we respectively obtain the following solutions:

$$v_{6,1}(x,t) = \frac{1}{2}(1 - \tanh(\theta)),$$

$$v_{6,2}(x,t) = \frac{1}{2}(1 - \coth(\theta)),$$

Where $\theta = \left(\frac{1}{2}k - \frac{1}{8}\right)t + \frac{1}{4}x.$

Case2.For $a_1 = 0$

For (S_7), since $\lambda^2 - 4\mu = 1 > 0$ so we have:

$$v_7(x,t) = \frac{-\frac{1}{4}\lambda^2 + \frac{1}{4}}{\frac{1}{2}\frac{-\upsilon_1 \sinh(\theta) + \upsilon_2 \cosh(\theta)}{\upsilon_1 \cosh(\theta) - \upsilon_2 \sinh(\theta)} - \frac{1}{2}\lambda} - \frac{1}{2}\lambda + \frac{1}{2}.$$

Where $\theta = \frac{1}{2}(k-1)t - \frac{1}{2}x.$

For (S_8), since $\lambda^2 - 4\mu = (k-1)^2 > 0$, so we have:

$$v_8(x,t) = \frac{\frac{1}{4}(-1-k)^2 - \frac{1}{4}\lambda^2 - k}{\frac{1}{2}\frac{\sqrt{(k-1)^2}\left(\upsilon_1 \sinh(\theta) + \upsilon_2 \cosh(\theta)\right)}{\upsilon_1 \cosh(\theta) + \upsilon_2 \sinh(\theta)} - \frac{1}{2}\lambda} - \frac{1}{2}\lambda + \frac{1}{2} + \frac{1}{2}k,$$

Where $\theta = \left(\frac{1}{2}k^2 - \frac{1}{2}\right)t + \left(\frac{1}{2}k - \frac{1}{2}\right)x.$

For (S_9), since $\lambda^2 - 4\mu = (k-1)^2 > 0$, so we have:

$$v_9(x,t) = \frac{\frac{1}{4}k^2 - \frac{1}{4}\lambda^2}{\frac{1}{2}\frac{k\left(\upsilon_1 \sinh(\theta) + \upsilon_2 \cosh(\theta)\right)}{\upsilon_1 \cosh(\theta) + \upsilon_2 \sinh(\theta)} - \frac{1}{2}\lambda} - \frac{1}{2}\lambda,$$

Where $\theta = \left(\frac{1}{2}k^2 - \frac{1}{2}k\right)t + \frac{1}{2}k\,x.$

For (S_{10}), since $\lambda^2 - 4\mu = \frac{1}{4} > 0$ so, we have

$$v_{10}(x,t) = \frac{\frac{1}{2}\lambda^2 - \frac{1}{8}}{\frac{1}{4}\frac{\upsilon_1 \sinh(\theta) + \upsilon_2 \cosh(\theta)}{\upsilon_1 \cosh(\theta) + \upsilon_2 \sinh(\theta)} - \frac{1}{2}\lambda} + \lambda + \frac{1}{2},$$

Where $\theta = \left(-\frac{1}{8} + \frac{1}{2}k\right)t + \frac{1}{4}x.$

For (S_{11}), since $\lambda^2 - 4\mu = \frac{(k-1)^2}{4} > 0$, we have

$$v_{11}(x,t) = \frac{-\frac{1}{2}\left(\frac{1}{2} + \frac{1}{2}k\right)^2 + \frac{1}{2}\lambda^2 + \frac{1}{2}k}{\frac{1}{4}\frac{(-1+k)\left(-\upsilon_1 \sinh(\theta) + \upsilon_2 \cosh(\theta)\right)}{\upsilon_1 \cosh(\theta) - \upsilon_2 \sinh(\theta)} - \frac{1}{2}\lambda} + \lambda + \frac{1}{2} + \frac{1}{2}k,$$

Where $\theta = -\left(-\frac{1}{8}k^2 + \frac{1}{8}\right)t - \left(\frac{1}{4}k - \frac{1}{4}\right)x.$

$\theta = -\left(-\frac{1}{8}k^2 + \frac{1}{8}\right)t - \left(\frac{1}{4}k - \frac{1}{4}\right)x.$

For (S_{12}), since $\lambda^2 - 4\mu = \frac{(k)^2}{4} > 0$,we have

$$v_{12}(x,t) = \frac{-\frac{1}{8}k^2 + \frac{1}{2}\lambda^2}{\frac{1}{4}\frac{k\left(-\upsilon_1 \sinh(\theta) + \upsilon_2 \cosh(\theta)\right)}{\upsilon_1 \cosh(\theta) - \upsilon_2 \sinh(\theta)} - \frac{1}{2}\lambda} + \frac{1}{2}k + \lambda,$$

Where $\theta = \frac{1}{8}k(k-4)t - \frac{1}{4}x\,k.$

For (S_{13}), since $\lambda^2 - 4\mu => 0$, wew have:

$$v_{13}(x,t) = -\frac{2}{\frac{1}{2}\frac{\upsilon_1 \sinh(\theta) + \upsilon_2 \cosh(\theta)}{\upsilon_1 \cosh(\theta) + \upsilon_2 \sinh(\theta)} + \frac{3}{2}} + 2 - \frac{\upsilon_1 \sinh(\theta) + \upsilon_2 \cosh(\theta)}{\upsilon_1 \cosh(\theta) + \upsilon_2 \sinh(\theta)},$$

By setting $\upsilon_2 = 0$ and $\upsilon_1 = 0$, we respectively obtain the following solutions:

$$v_{13,1}(x,t) = -\frac{2}{\frac{1}{2}\tanh(\theta)+\frac{3}{2}} + 2 - \tanh(\theta),$$

$$v_{13,2}(x,t) = -\frac{2}{\frac{1}{2}\coth(\theta)+\frac{3}{2}} + 2 - \coth(\theta),$$

Where $\theta = 3t + \frac{1}{2}x.$

For (S_{14}), since $\lambda^2 - 4\mu => 0$, we have:

$$v_{14}(x,t) = -\frac{1}{\frac{3}{2}\frac{\upsilon_1\sinh(\theta)+\upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta)+\upsilon_2\sinh(\theta)} - \frac{9}{2}} + \frac{5}{6} - \frac{1}{6}\frac{\upsilon_1\sinh(\theta)+\upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta)+\upsilon_2\sinh(\theta)},$$

By setting $\upsilon_2 = 0$and $\upsilon_1 = 0$, we respectively obtain the following solutions:

$$v_{14,1}(x,t) = -\frac{1}{\frac{3}{2}\tanh(\theta)-\frac{9}{2}} + \frac{5}{6} - \frac{1}{6}\tanh(\theta),$$

$$v_{14,2}(x,t) = -\frac{1}{\frac{3}{2}\coth(\theta)-\frac{9}{2}} + \frac{5}{6} - \frac{1}{6}\coth(\theta),$$

$$\theta = \frac{1}{72}t + \frac{1}{12}x.$$

For (S_{15}), since $\lambda^2 - 4\mu => 0$,we have:

$$v_{15}(x,t) = -\frac{1}{\frac{3}{2}\frac{\upsilon_1\sinh(\theta)+\upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta)+\upsilon_2\sinh(\theta)} + \frac{9}{2}} + \frac{1}{6} - \frac{1}{6}\frac{\upsilon_1\sinh(\theta)+\upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta)+\upsilon_2\sinh(\theta)},$$

By setting $\upsilon_2 = 0$ and $\upsilon_1 = 0$, we respectively obtain the following solutions:

$$v_{15,1}(x,t) = -\frac{1}{\frac{3}{2}\tanh(\theta)+\frac{9}{2}} + \frac{1}{6} - \frac{1}{6}\tanh(\theta),$$

$$v_{15,2}(x,t) = -\frac{1}{\frac{3}{2}\coth(\theta)+\frac{9}{2}} + \frac{1}{6} - \frac{1}{6}\coth(\theta),$$

Where $\theta = \frac{5}{72}t + \frac{1}{12}x.$

For (S_{16}), since $\lambda^2 - 4\mu => 0$, we have:

$$v_{16}(x,t) = -\frac{2}{\frac{1}{2}\frac{-\upsilon_1\sinh(\theta)+\upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta)-\upsilon_2\sinh(\theta)}-\frac{3}{2}} - 1 - \frac{-\upsilon_1\sinh(\theta)+\upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta)-\upsilon_2\sinh(\theta)}$$

By setting $\upsilon_2 = 0$ and $\upsilon_1 = 0$, we respectively obtain the following solutions:

$$v_{16,1}(x,t) = -\frac{2}{-\frac{1}{2}\tanh(\theta)-\frac{3}{2}} - 1 + \tanh(\theta),$$

$$v_{16,2}(x,t) = -\frac{2}{-\frac{1}{2}\coth(\theta)-\frac{3}{2}} - 1 + \coth(\theta),$$

Where $\theta = \frac{5}{2}t - \frac{1}{2}x.$

For (S_{17}), since $\lambda^2 - 4\mu => 0$, we have:

$$v_{17}(x,t) = \frac{4}{\frac{7}{2}\frac{\upsilon_1\sinh(\theta)+\upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta)+\upsilon_2\sinh(\theta)}-\frac{21}{2}} + \frac{1}{2} + \frac{1}{14}\frac{\upsilon_1\sinh(\theta)+\upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta)+\upsilon_2\sinh(\theta)},$$

By setting $\upsilon_2 = 0$ and $\upsilon_1 = 0$, we respectively obtain the following solutions:

$$v_{17,1}(x,t) = \frac{4}{\frac{7}{2}\tanh(\theta)-\frac{21}{2}} + \frac{1}{2} + \frac{1}{14}\tanh(\theta),$$

$$v_{17,2}(x,t) = \frac{4}{\frac{7}{2}\coth(\theta)-\frac{21}{2}} + \frac{1}{2} + \frac{1}{14}\coth(\theta),$$

For (S_{18}), since $\lambda^2 - 4\mu => 0$, we have:

$$v_{18}(x,t) = \frac{4}{\frac{1}{2}\frac{\upsilon_1\sinh(\theta)+\upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta)+\upsilon_2\sinh(\theta)}-\frac{3}{2}} + \frac{7}{2} + \frac{1}{2}\frac{\upsilon_1\sinh(\theta)+\upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta)+\upsilon_2\sinh(\theta)},$$

By setting $\upsilon_2 = 0$ and $\upsilon_1 = 0$, we respectively obtain the following solutions:

$$v_{18,1}(x,t) = \frac{4}{\frac{1}{2}\tanh(\theta) - \frac{3}{2}} + \frac{7}{2} + \frac{1}{2}\tanh(\theta),$$

$$v_{18,2}(x,t) = \frac{4}{\frac{1}{2}\coth(\theta) - \frac{3}{2}} + \frac{7}{2} + \frac{1}{2}\coth(\theta),$$

For (S_{19}), since $\lambda^2 - 4\mu => 0$, we have:

$$v_{19}(x,t) = \frac{1}{\frac{3}{4}\frac{\upsilon_1\sinh(\theta) + \upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta) + \upsilon_2\sinh(\theta)} + \frac{9}{4}} + \frac{7}{12} + \frac{1}{12}\frac{\upsilon_1\sinh(\theta) + \upsilon_2\cosh(\vartheta)}{\upsilon_1\cosh(\theta) + \upsilon_2\sinh(\theta)},$$

By setting $\upsilon_2 = 0$ and $\upsilon_1 = 0$, we respectively obtain the following solutions:

$$v_{19,1}(x,t) = \frac{1}{\frac{3}{4}\tanh(\theta) + \frac{9}{4}} + \frac{7}{12} + \frac{1}{12}\tanh(\theta),$$

$$v_{19,2}(x,t) = \frac{1}{\frac{3}{4}\coth(\theta) + \frac{9}{4}} + \frac{7}{12} + \frac{1}{12}\coth(\theta),$$

Where $\theta = \frac{1}{72}t + \frac{1}{12}x.$

For (S_{20}), since $\lambda^2 - 4\mu => 0$, we have:

$$v_{20}(x,t) = \frac{1}{\frac{3}{4}\frac{\upsilon_1\sinh(\theta) + \upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta) + \upsilon_2\sinh(\theta)} - \frac{9}{4}} + \frac{5}{12} + \frac{1}{12}\frac{\upsilon_1\sinh(\theta) + \upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta) + \upsilon_2\sinh(\theta)},$$

By setting $\upsilon_2 = 0$ and $\upsilon_1 = 0$, we respectively obtain the following solutions:

$$v_{20,1}(x,t) = \frac{1}{\frac{3}{4}\tanh(\theta) - \frac{9}{4}} + \frac{5}{12} + \frac{1}{12}\tanh(\theta),$$

$$v_{20,2}(x,t) = \frac{1}{\frac{3}{4}\coth(\theta) - \frac{9}{4}} + \frac{5}{12} + \frac{1}{12}\coth(\theta),$$

For (S_{21}), since $\lambda^2 - 4\mu => 0$, we have:

$$v_{21}(x,t)=\frac{-4}{\frac{1}{2}\frac{\upsilon_1\sinh(\theta)-\upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta)-\upsilon_2\sinh(\theta)}+\frac{3}{2}}-\frac{5}{2}+\frac{1}{2}\frac{-\upsilon_1\sinh(\theta)+\upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta)-\upsilon_2\sinh(\theta)},$$

By setting $\upsilon_2=0$ and $\upsilon_1=0$, we respectively obtain the following solutions:

$$v_{21,1}(x,t)=\frac{4}{-\frac{1}{2}\tanh(\theta)+\frac{3}{2}}-\frac{5}{2}-\frac{1}{2}\tanh(\theta),$$

$$v_{21,2}(x,t)=\frac{4}{-\frac{1}{2}\coth(\theta)+\frac{3}{2}}-\frac{5}{2}-\frac{1}{2}\coth(\theta),$$

Where $\theta=\frac{5}{2}t-\frac{1}{2}x.$

For (S_{22}), since $\lambda^2-4\mu=>0$, we have:

$$v_{22}(x,t)=\frac{4}{\frac{7}{2}\frac{\upsilon_1\sinh(\theta)+\upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta)+\upsilon_2\sinh(\theta)}+\frac{21}{2}}+\frac{1}{2}+\frac{1}{14}\frac{\upsilon_1\sinh(\theta)+\upsilon_2\cosh(\theta)}{\upsilon_1\cosh(\theta)+\upsilon_2\sinh(\theta)},$$

By setting $\upsilon_2=0$ and $\upsilon_1=0$, we respectively obtain the following solutions:

$$v_{22,1}(x,t)=\frac{4}{\frac{7}{2}\tanh(\theta)+\frac{21}{2}}+\frac{1}{2}+\frac{1}{14}\tanh(\theta),$$

$$v_{22,2}(x,t)=\frac{4}{\frac{7}{2}\coth(\theta)+\frac{21}{2}}+\frac{1}{2}+\frac{1}{14}\coth(\theta),$$

Where $\theta=\frac{1}{98}t+\frac{1}{14}x.$

Note: It is observed that all first-order particular solutions correspond to kink-type solutions, whereas the second-order solutions represent traveling wave solutions.

4- Drawings to illustrate the behavior of some proposed solutions

$$> v_{1,1}:=\frac{1}{2}-\frac{1}{2}\tanh\left(\frac{1}{2}t-\frac{1}{2}x\right)$$

$$v_{1,1}:=\frac{1}{2}-\frac{1}{2}\tanh\left(\frac{1}{2}t-\frac{1}{2}x\right)$$

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> plot3d($v_{1,1}$, x = −2 ..2, t = −2 ..2)

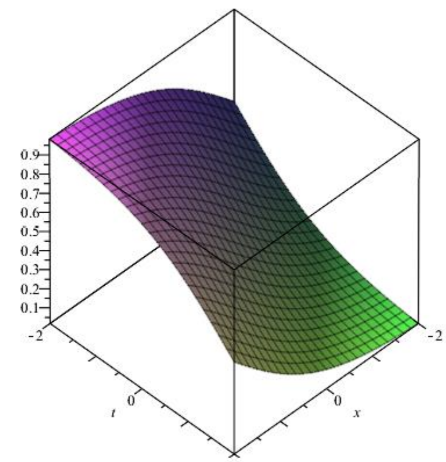

Figure 1. Kink solution of $v_{1,1}(x,t)$ for $k = 2, x = -2..2, t = -2..2$

> $v_{1,2} := \frac{1}{2} - \frac{1}{2}\coth\left(\frac{1}{2}t - \frac{1}{2}x\right)$

$v_{1,2} := \frac{1}{2} - \frac{1}{2}\coth\left(\frac{1}{2}t - \frac{1}{2}x\right)$

> plot3d($v_{1,2}$, x = −2 ..2, t = −2 ..2)

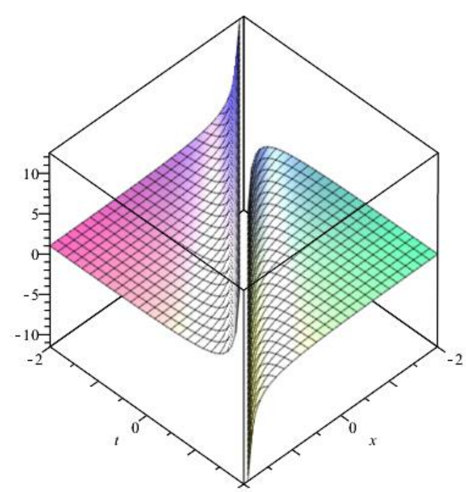

Figure 2. Traveling wave solution of $v_{1,2}(x,t)$ for $k = 2, x = -2..2, t = -2..2$

$$v_{7,1} := -\frac{3}{4\left(-\frac{1}{2}\frac{\sinh\left(\frac{1}{2}t - \frac{1}{2}x\right)}{\cosh\left(\frac{1}{2}t - \frac{1}{2}x\right)} - 1\right)} - \frac{1}{2}$$

>

$$v_{7,1} := -\frac{3}{-\frac{2\sinh\left(\frac{1}{2}t-\frac{1}{2}x\right)}{\cosh\left(\frac{1}{2}t-\frac{1}{2}x\right)}-4}-\frac{1}{2}$$

>**plot3d**$\left(v_{7,1}, x=-2..2, t=-2..2\right)$

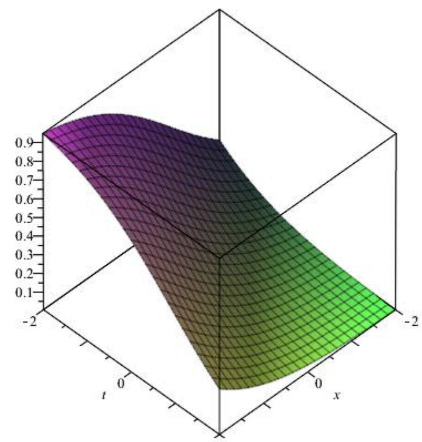

Figure 3.Kink solution of $v_{7,1}(x,t)$ for $k = 2, x = -2..2, t = -2..2$

$$v_{7,2} := -\frac{3}{4\left(-\frac{1}{2}\frac{\cosh\left(\frac{1}{2}t-\frac{1}{2}x\right)}{\sinh\left(\frac{1}{2}t-\frac{1}{2}x\right)}-1\right)}-\frac{1}{2}$$

>

$$v_{7,2} := -\frac{3}{-\frac{2\cosh\left(\frac{1}{2}t-\frac{1}{2}x\right)}{\sinh\left(\frac{1}{2}t-\frac{1}{2}x\right)}-4}-\frac{1}{2}$$

>**plot3d**$\left(v_{7,2}, x=-2..2, t=-2..2\right)$

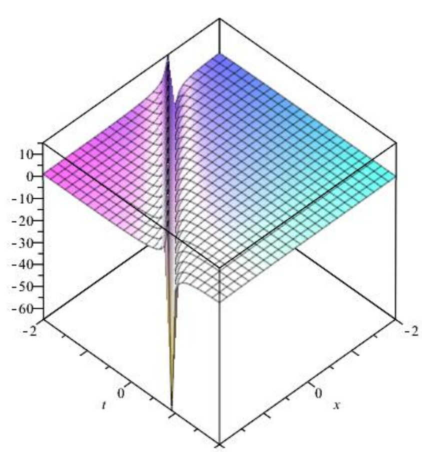

Figure 4. Traveling wave solution of $v_{7,2}(x,t)$ for $k = 2, x = -2..2, t = -2..2$

$$v_{13,1} := -\frac{2}{\frac{1}{2}\frac{\sinh\left(3t+\frac{1}{2}x\right)}{\cosh\left(3t+\frac{1}{2}x\right)}+\frac{3}{2}} + 2 - \frac{\sinh\left(3t+\frac{1}{2}x\right)}{\cosh\left(3t+\frac{1}{2}x\right)}$$

>

$$v_{13,1} := -\frac{2}{\frac{1}{2}\frac{\sinh\left(3t+\frac{1}{2}x\right)}{\cosh\left(3t+\frac{1}{2}x\right)}+\frac{3}{2}} + 2 - \frac{\sinh\left(3t+\frac{1}{2}x\right)}{\cosh\left(3t+\frac{1}{2}x\right)}$$

>plot3d($v_{13,1}$, x = −2 ..2, t = −2 ..2)

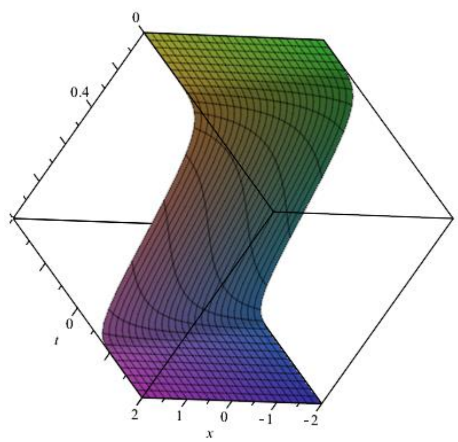

Figure 5.Kink solution of $v_{13,1}(x,t)$ for $\lambda = 2, k = 2, x = -2..2, t = -2..2$

$$v_{13,2} := -\frac{2}{\frac{1}{2}\frac{\cosh\left(3t+\frac{1}{2}x\right)}{\sinh\left(3t+\frac{1}{2}x\right)}+\frac{3}{2}} + 2 - \frac{\cosh\left(3t+\frac{1}{2}x\right)}{\sinh\left(3t+\frac{1}{2}x\right)}$$

>

$$v_{13,2} := -\frac{2}{\frac{1}{2}\frac{\cosh\left(3t+\frac{1}{2}x\right)}{\sinh\left(3t+\frac{1}{2}x\right)}+\frac{3}{2}} + 2 - \frac{\cosh\left(3t+\frac{1}{2}x\right)}{\sinh\left(3t+\frac{1}{2}x\right)}$$

>plot3d($v_{13,2}$, x = −2 ..2, t = −2 ..2)

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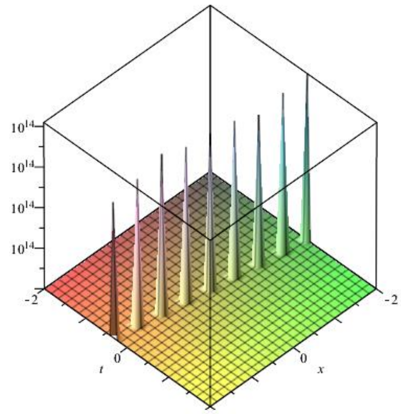

Figure 6. Traveling wave solution of $v_{13,2}(x,t)$ for $\lambda = 2, k = 2, x = -2..2, t = -2..2$

5- Conclusions

In this study, exact solitary and analytical solutions of the Burgers–Huxley equation have been successfully derived by employing the extended -expansion method combined with the traveling wave transformation. The results clearly demonstrate the effectiveness and robustness of the proposed approach in handling nonlinear partial differential equations arising in physics, mathematics, and engineering. The solitary wave solutions obtained highlight the method's capability to capture localized and stable structures, while the kink-type solutions describe transition waves connecting distinct asymptotic states.

Furthermore, the solutions presented herein can be regarded as a generalization of the results reported in References (Ebiwareme, 2021) and (A. M. Wazwaz, 2019)which appear as special cases within the broader framework established by this work. Importantly, the extended -expansion method can be viewed as a natural generalization of the classical tanh–coth method, thereby enriching the analytical toolbox available for nonlinear evolution equations. Several of the solutions reported here have not been previously documented in the literature, underscoring the novelty and efficiency of the extended -expansion method in constructing diverse families of exact solutions

6- References

- Abdoon, M. A. (2015). First Integral Method: A General Formula for Nonlinear Fractional Klein-Gordon Equation Using Advanced Computing Language. June, 127–134.
- Al-shawba, A. A., Abdullah, F. A., Gepreel, K. A., & Azmi, A. (2018). Solitary and periodic wave solutions of higher-dimensional conformable time-fractional differential equations using the (G. Adv Differ Equ). https://doi.org/10.1186/s13662-018-1814-5
- Ebiwareme, L. (2021). Banach Contraction Method and Tanh-coth Approach for the Solitary and Exact Solutions of Burger-Huxley and Kuramoto-Sivashinsky Equations. International *Journal of Mathematics Trends and Technology*, 67(4), 31–46. https://doi.org/10.14445/22315373/ijmtt-v67i4p506
- El-borai, M. M., & Al-masroub, R. M. (2015). Exact Solutions for Some Nonlinear Parabolic Equations. 10, 106–122.
- El-borai, M. M., El-sayed, W. G., & Al-masroub, R. M. (2015). Exact Solutions for Time Fractional Coupled Whitham-Broer-Kaup Equations Via Exp-Function Method. *IRJET*, 02(06), 307–315.
- El-borai, M. M., El-sayed, W. G., & Al-masroub, R. M. (2016). Exact Solutions of Some Nonlinear Partial Differential Equations via Extended Method. 36(1), 60–71.
- Fan, E. (2000). Extended tanh-function method and its applications to nonlinear equations. *Physics Letters A*, 277(4–5), 212--218.
- Gepreel, K. A. (2014). Explicit Jacobi elliptic exact solutions for nonlinear partial fractional differential equations. 1–14.
- He, J. (2013). Exp-function Method for Fractional Differential Equations. 14(6), 363–366. https://doi.org/10.1515/ijnsns-2011-0132
- Kumar, H., & Chand, F. (2014). Exact traveling wave solutions of some nonlinear evolution equations. 1–10. https://doi.org/10.1007/s40094-014-0114-z
- Li, W., Yang, H., & He, B. (2014). Exact Solutions of Fractional Burgers and Cahn-Hilliard Equations Using Extended Fractional Riccati Expansion Method. 2014(2).
- Maruno, K., Ankiewicz, A., & Akhmediev, N. (2003). Exact localized and periodic solutions of the discrete complex Ginzburg – Landau equation. 221, 199–209. https://doi.org/10.1016/S0030-4018(03)01457-3
- Maurya, D. K., Singh, R., & Rajoria, Y. K. (2019). A mathematical model to solve the Burgers-Huxley equation by using New Homotopy Perturbation Method. *International Journal of Mathematical, Engineering and Management Sciences,* 4(6), 1483–1495. https://doi.org/10.33889/IJMEMS.2019.4.6-117

−Parkes, E. J. (2009). Comments on the use of the tanh-coth expansion method for finding solutions to nonlinear evolution equations ∗. 1–11. https://doi.org/10.1016/j.amc.2009.11.037.

−Proceedings, A. I. P. C., El, A., & Academy, S. (2016). Applications of the functional variable method for finding the exact solutions of nonlinear evolution equations in mathematical physics. January. https://doi.org/10.1063/1.4825916

−Wazwaz, A. (2007). The tanh – coth method for solitons and kink solutions for nonlinear parabolic equations. 188, 1467–1475. https://doi.org/10.1016/j.amc.2006.11.013

−Wazwaz, A. M. (2019). Partial Differential Equations and Solitary Waves Theory. In *Springer Berlin Heidelberg* (Vol. 53, Issue 9).

−Zayed, E. M. E., & Alurrfi, K. A. E. (2014). Solving Two Higher Order Nonlinear Evolution Equations. 2014(2).

−Zayed, E. M. E., & Alurrfi, K. A. E. (2015). The modified Kudryashov method for solving some seventh order nonlinear PDEs in mathematical physics ∗. 11(4), 308–319.

−Zerarka, A., Ouamane, S., & Attaf, A. (2010). On the functional variable method for finding exact solutions to a class of wave equations. *Applied Mathematics and Computation*, 217(7), 2897–2904. https://doi.org/10.1016/j.amc.2010.08.070

−Zheng, B. (2013). Exact solutions for some fractional partial differential equations by the (G ′ / G) method. *Mathematical Problems in Engineering*, 2013(August), 0–5. https://doi.org/10.1155/2013/826369